 BCA(I)-103

2018

Time : 3Hrs *Full Marks : 80*

Candidates are required to give their answers in their own words as far as practicable.

The questions are of equal value.

Answer any five questions in which Q. No.1 is compulsory.

1. Choose the correct answer from the given alternatives:

(a) $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}$ = ...

(i) $\frac{1}{e}$ (ii) e^{x}

(iii) L (iv) None of these

(b) The domain and range of $f(x)=\sqrt{4-x^2}$ are given by

(i) $(-\infty,\infty),[0,2]$ (ii) $[-2,2],(0,\infty)$

(iii) [-2, 2], [0, 2] (iv) None of these

(c) If $y=a^x$, then $\frac{dy}{dx}$ = ...

(i) $a^x.\log a$ (ii) $x^a.\log x$

(iii) $x.a^{x-1}$ (iv) None of these

P.T.O.

(d) $\frac{d}{d(\sin x)}\{e^{\tan x}\} = \ldots\ldots$

(i) $e^{\tan x}.\sec^2 x$ (ii) $\sec^3 x.e^{\tan x}$

(iii) $\sec^3 x.e^{\sec x}$ (iv) None of these

(e) At $x = 2$, $f(x) = [x]$ is

(i) Continuous but not differentiable

(ii) Differentiable but not continuous

(iii) Continuous as well as differentiable

(iv) None of these

(f) If $x = at^2$, $y = 2at$, then $\frac{dy}{dx} = \ldots\ldots$

(i) $\frac{1}{t}$ (ii) $-\frac{1}{t^2}$

(iii) $\frac{-2}{t}$ (iv) None of these

(g) $\int \frac{dx}{q + r^2} = \ldots\ldots$

(i) $\tan^{-1}\frac{x}{3} + c$ (ii) $\frac{1}{3}\tan^{-1}\frac{x}{3}$

(iii) $3\tan^{-1}\frac{x}{3} + c$ (iv) None of these

(h) If $f(x, y) = 2x^2 - xy + 2y^2$, then $\frac{\partial t}{\partial n}\, at(1, 2)$ is

(i) 8 (ii) -7

(iii) 14 (iv) None of these

2. Answer any two of the following:

Prove that

(a) $\lim_{x\to 0}\left(\frac{\tan x - \sin x}{x^3}\right) = \frac{1}{2}$

(b) $\lim_{x\to 0}\frac{a^x - 1}{x} = \log_e a$

(c) $\lim_{x\to 0}\frac{\log(1+x)}{x} = 1$

3. Answer any two of the following:

(a) If $y = \frac{1 + x - 4\sqrt{x}}{x}$, find $\frac{dy}{dx}$.

(b) Differentiate : $\log[\log(\log x)]$

(c) Find $\frac{dy}{dx}$, when $y = (\cos x)^{\cos x}$

4. Evaluate any two of the following :

(a) $\int \frac{dx}{\sqrt{a^2 - x^2}}$

(b) $\int_0^{1/4} \sqrt{1 - \sin 2x}\, dx$

(c) $\int \frac{(x-1)}{(x+1)(x-2)} dx$

5. (a) Find the maximum and minimum values of the function

$2x^3 - 21x^2 + 36x - 20$

(b) Find the maximum value of

$$\frac{x^2 - 2x + 4}{x^2 + 2x + 4}$$

6. (a) $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$ if $u = \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1$

(b) Find $\frac{du}{dx}$ if $u = \text{Sin}(x^2 + y^2)$ where

$a^2x^2 + b^2y^2 = c^2$

7. (a) State and prove Lagrange's interpolation formula.

(b) Find first and second derivatives of the function given below at the point $x = 1.2$

x	1	2	3	4	5
y	0	1	5	6	5

8. (a) Verify Rolle's theorem in the case of the following function:

$f(x) = 2x^3 + x^2 - 4x - 2$

(b) Find the expansion of e^x using Taylor Series.

9. (a) Find the whole area of the circle $x^2 + y^2 = a^2$

(b) Find the length of the arc of the parabola $y^2 = 4ax$ cut off by the line $y = 2x$.

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